ROLE OF INTERNAL UNSTEADY PROCESSES IN PROBLEMS OF THE MOTION OF TWO-PHASE STREAMS

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A model of a two-phase stream is developed which allows one to calculate the basic hydraulic and continuity characteristics of the stream as well as the mode of flow on the basis of the allowance for the unsteadiness of the motion of the separate phases.

The existing analytical models of the flow of a two-phase stream do not take into account one of its most important properties – the unsteadiness of the motion of the separate phases [1-13].

This unsteadiness is manifested in the appearance of pulsations, stable and very substantial in size, in the pressure, frictional shear stress at the channel walls, and density of the two-phase stream [14-21].

The one-dimensional, adiabatic, stabilized flow of a two-phase stream through channels of constant cross section or local resistances is under discussion. The motion of the two-phase mixture is assumed to be pressurized and high-velocity but subsonic. The compressibility of the medium is not taken into account, which is valid when $\Delta\bar{P}/\bar{P} \ll 1$.

The real two-phase stream is replaced by a quasihomogeneous stream but with time-varying velocity w_{mi} and density ρ_{mi} of the mixture, which are determined by the expressions

$$w_{mi}(\tau) = \frac{G_{mi}(\tau)}{Fg\rho_{mi}(\tau)} = w_0(\tau)\,[1 + x(\tau)\,\gamma_0], \tag{1}$$

$$\rho_{mi}(\tau) = \rho_1\,[1-\beta(\tau)] + \rho_2\beta(\tau). \tag{2}$$

Another basic assumption consists in the possibility of calculating the instantaneous values of ΔP in local resistances or of $(-\partial P/\partial z)$ in channels of constant cross section using the well-known quadratic equations of the following type:

$$\Delta P = \zeta\,\frac{\rho_{mi}(\tau)\,w_{mi}^2(\tau)}{2}, \tag{3}$$

$$\left(-\frac{\partial P}{\partial z}\right) = \xi\,\frac{\rho_{mi}(\tau)\,w_{mi}^2(\tau)}{2d}. \tag{4}$$

Since we are considering high-velocity flow, it is assumed that the hydraulic resistance coefficients ζ and ξ are constants which do not depend on the stream velocity w_{mi} or the time τ. Since

$$w_0 = \frac{G_{mi}}{g\rho_1 F};\quad x = \frac{G_2}{G_{mi}};\quad \gamma_0 = \frac{\rho_1-\rho_2}{\rho_2};\quad G_{mi} = G_1 + G_2, \tag{5}$$

Eq. (3) takes the form

$$\Delta P(\tau) = \frac{\zeta}{2g^2\rho_1 F^2}\,[G_1^2(\tau) + (1+\gamma_0)\,G_2^2(\tau) + (2+\gamma_0)\,G_1(\tau)\,G_2(\tau)]. \tag{6}$$

Henceforth the averaging of any basic time-varying characteristic of the two-phase stream (x, φ, β, ρ_{mi}, ΔP, w_i) will be performed as follows:

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$$\bar{\chi} = \frac{1}{T} \int_0^T \chi(\tau)\, d(\tau). \tag{7}$$

For the periodic functions T is the oscillation period of the variable quantity.

The application of the averaging rule (7) to Eq. (6) gives the following result:

$$\Delta\bar{P} = \frac{\zeta}{2g^2\rho_1 F^2} [(\overline{G_1^2}) + (1+\gamma_0)(\overline{G_2^2}) + (2+\gamma_0)(\overline{G_1 G_2})]. \tag{8}$$

In accordance with the steady homogeneous model [3, 10] of a two-phase stream the calculated pressure losses are determined by the following equation:

$$\Delta P_{hom} = \frac{\zeta}{2g^2\rho_1 F^2} (\bar{G}_{mi})^2 (1 + \bar{x}_f \gamma_0), \tag{9}$$

where the weight flow-rate vapor content is calculated from the standard dependence

$$\bar{x}_f = \frac{\bar{G}_2}{\bar{G}_{mi}}. \tag{10}$$

On the basis of Eqs. (9) and (10) we obtain

$$\Delta P_{hom} = \frac{\zeta}{2g^2\rho_1 F^2} [(\bar{G}_1)^2 + (1+\gamma_0)(\bar{G}_2)^2 + (2+\gamma_0)\bar{G}_1 \bar{G}_2]. \tag{11}$$

Equations (8) and (11) make it possible to obtain an expression for the ratio of the pressure drop in an unsteady two-phase stream to that calculated from a steady homogeneous model for local resistances:

$$\Psi = \frac{\Delta\bar{P}}{\Delta P_{hom}} = \frac{\overline{G_1^2} + (1+\gamma_0)\overline{G_2^2} + (2+\gamma_0)(\overline{G_1 G_2})}{(\bar{G}_1)^2 + (1+\gamma_0)(\bar{G}_2)^2 + (2+\gamma_0)\bar{G}_1\bar{G}_2}. \tag{12}$$

We obtain exactly the same expression for the coefficient Ψ in an analysis of Eq. (4) for the unsteady motion of a two-phase stream in a straight channel of constant cross section.

By definition, the dimensionless coefficient Ψ fully coincides with the so-called coefficient of inhomogeneity of a two-phase stream [9, 10].

In the case of the absence of pulsations of the phase flow rates with time, when $(\bar{G}_i)^2 = \overline{G_i^2}$ and $\overline{G_1 G_2} = \bar{G}_1 \bar{G}_2$, the coefficient Ψ equals 1.0 according to (12).

In all other cases the quantity Ψ can be either larger or smaller than $\Psi = 1.0$ depending on the nature of the pulsations of the phase flow rates with time.

With one-dimensional flow of a mixture in a local resistance the instantaneous value of the power is determined by the expression

$$N(\tau) = \Delta P(\tau)\, w_{mi}(\tau)\, F = \zeta \frac{G_{mi}^3(\tau)\,[1 + x(\tau)\gamma_0]^2}{2g^3\rho_1 F^2}. \tag{13a}$$

Similarly, for the power of a two-phase stream in a channel of constant cross section, taken per unit length of the channel, we have

$$\frac{\partial N}{\partial z} = \left(-\frac{\partial P}{\partial z}\right) w_{mi} F = \frac{\xi}{2g^3\rho_1 F^2 d} G_{mi}^3 (1 + x\gamma_0)^2. \tag{13}$$

On the basis of Eqs. (7), (13a), and (13) we have for the time-averaged powers of an unsteady stream

$$\bar{N} = \frac{\zeta}{2g^3 F^2 \rho_1} \overline{G_{mi}^3 (1 + x\gamma_0)^2}, \tag{14}$$

$$\frac{\partial \bar{N}}{\partial z} = \frac{\xi}{2g^3F^2\rho_1 d}\,\overline{G^3_{mi}(1+x\gamma_0)^2}. \tag{15}$$

The corresponding powers of a steady, homogeneous, two-phase stream have the form

$$N_{hom} = \frac{\zeta}{2g^3F^2\rho_1}(\bar{G}_{mi})^3(1+\bar{x}_f\gamma_0)^2, \tag{16}$$

$$\frac{\partial N_{hom}}{\partial z} = \frac{\xi}{2g^3F^2\rho_1 d}(\bar{G}_{mi})^3(1+\bar{x}_f\gamma_0)^2. \tag{17}$$

Dividing Eq. (14) by Eq. (16) or (15) by (17), we obtain the following expression for the ratio of the powers of unsteady and steady, homogeneous, two-phase streams:

$$E = \frac{\overline{G^3_{mi}(1+x\gamma_0)^2}}{(\bar{G}_{mi})^3(1+\bar{x}_p\gamma_0)^2}. \tag{18}$$

With allowance for Eqs. (5) and (10) the expression (18) takes the following form:

$$E = \frac{\overline{G_1^3} + (1+\gamma_0)^2\,\overline{G_2^3} + (3+2\gamma_0)\,\overline{G_1^2G_2} + (1+\gamma_0)(3+\gamma_0)\,\overline{G_1G_2^2}}{(\bar{G}_1)^3 + (1+\gamma_0)^2(\bar{G}_2)^3 + (3+2\gamma_0)(\bar{G}_1)^2\bar{G}_2 + (1+\gamma_0)(3+\gamma_0)\bar{G}_1(\bar{G}_2)^2}. \tag{19}$$

Depending on the character of the pulsations in the flow rates G_i of the phases with time the coefficient E, like Ψ, can be larger than, less than, or equal to unity (the latter occurs when G_i = const). If the coefficient Ψ characterizes the degree of inhomogeneity of an unsteady two-phase stream from the point of view of pressure losses in the stream, then the coefficient E is the analogous energetic characteristic of an unsteady two-phase stream.

The character of the time variation in the flow rates G_i of the phases is not known a priori. At present it does not seem possible to establish it directly by experimental means.

In this connection two types of periodic oscillations of the phase flow rates with time are analyzed below: the type of rectangular pulses (sharply expressed pulsations in the phase flow rates) and sinusoidal oscillations (smoothed harmonic variations in the phase flow rates with time).

The mathematical notation for the first type of oscillations have the following form:

$$\left.\begin{aligned} &\text{for } 0<\tau<\tau_1 \quad G_1 = G_{11} = \text{const}_1; \quad G_2 = G_{21} = \text{const}_2, \\ &G_{mi} = G_{mi_1} = G_{11} + G_{21} = \text{const}_3, \\ &\text{for } \tau_1<\tau<(\tau_1+\tau_2) \quad G_1 = G_{12} = \text{const}_4; \quad G_2 = G_{22} = \text{const}_5, \\ &G_{mi} = G_{mi_2} = G_{12}G_{22} = \text{const}_6. \end{aligned}\right\} \tag{20}$$

The pulse amplitudes G_{ij} and their duration τ_i, just like the total oscillation period $T = \tau_1 + \tau_2$, are completely arbitrary.

The instantaneous values of the weight vapor content x_i accordingly also vary abruptly with time:

$$\begin{aligned} &\text{for } 0<\tau<\tau_1 \quad x_1 = \frac{G_{21}}{G_{11}+G_{21}}, \\ &\text{for } \tau_1<\tau<T \quad x_2 = \frac{G_{22}}{G_{12}+G_{22}}. \end{aligned} \tag{21}$$

The mathematical notation for the harmonic type of oscillations in the phase flow rates with arbitrary amplitudes, frequencies, and phase shifts in time have the form

$$\left.\begin{aligned} G_1 &= G_{10}\,[1 + n\sin(\omega_1\tau + \lambda_1)], \\ G_2 &= G_{20}\,[1 + m\sin(\omega_2\tau + \lambda_2)]. \end{aligned}\right\} \tag{22}$$

It is obvious that

$$\bar{G}_1 = G_{10}, \quad \bar{G}_2 = G_{20} \text{ and } \bar{x}_f = \frac{G_{20}}{G_{10}+G_{20}}. \tag{23}$$

To determine the concrete values of the dynamic and average hydraulic characteristics of the unsteady two-phase stream under consideration one assumes that there is a minimum in the energy losses in this stream. The principle of a kinetic energy minimum or a minimum in the entropy rise in a steady two-phase stream has been used successfully by a number of authors [22-24] in the theoretical analysis of the critical discharge of a two-phase mixture through nozzles and pipes.

From the mathematical point of view the application of the principle of a minimum in energy losses to the unsteady flow of a two-phase stream comes down to a search for the minimum value of the energetic parameter E of (19) for fixed values of γ_0, $\bar{G}_1$, and $\bar{G}_2$ (or γ_0 and $\bar{x}_f$) with allowance for the concrete form of the oscillations in the phase flow rates determined by Eqs. (20) or (22). As a result of the minimization of the power E of a two-phase stream having oscillations in the phase flow rates of the rectangular pulse type (20) in the regions of variation $0 \le \bar{x}_f \le 1.0$ and $0 < \gamma_0 < +\infty$, one obtains the following:

$$\begin{gathered} \text{for } 0<\tau<\tau_1 \quad x_1 = 0 \text{ (flow of gas or vapor)}, \\ \text{for } \tau_1<\tau<(\tau_1+\tau_2) \quad x_2 = 1{,}0 \text{ (flow of liquid)}, \\ \frac{\tau_2}{\tau_1} = \frac{(1+\gamma_0)^{2/3}\,\bar{x}_f}{(1-\bar{x}_f)}. \end{gathered} \tag{24}$$

In this case the expression (12) for the coefficient of inhomogeneity Ψ takes the form

$$\Psi = \frac{\{1+\bar{x}_f\,[(1+\gamma_0)^{2/3}-1]\}\,\{1+\bar{x}_f\,[(1+\gamma_0)^{1/3}-1]\}}{1+\gamma_0\bar{x}_f}. \tag{25}$$

Correspondingly, the following results are obtained for the harmonic law (22):

$$\left.\begin{aligned} &\text{in region } 0<\bar{x}_f<1.0 \quad \omega_1=\omega_2=\omega, \quad \lambda_1-\lambda_2=\pi, \\ &\text{in region I } 0\leqslant \bar{x}_f \leqslant \bar{x}_{bo_1} = \frac{\sqrt{3(\gamma_0+1)(\gamma_0+3)}-3}{\gamma_0(\gamma_0+4)}, \\ &m=1.0;\ n=\frac{\bar{x}_f\,[3+2\gamma_0\bar{x}_f+\gamma_0(2+\gamma_0\bar{x}_f)]}{(1-\bar{x}_f)(3+2\gamma_0\bar{x}_f)}, \\ &\Psi=\Psi_I=1-\frac{(\bar{x}_f\gamma_0)^2(2+\gamma_0\bar{x}_f)}{2(3+2\bar{x}_f\gamma_0)^2}; \\ &\text{in region II } \bar{x}_{bo_1}\leqslant \bar{x}_f \leqslant \bar{x}_{bo_2} = \\ &=\frac{(1+\gamma_0)\sqrt{3(3+2\gamma_0)}-(3+2\gamma_0)}{\gamma_0(4+3\gamma_0)}, \\ &m=1.0; \quad n=1.0, \quad \Psi=\Psi_{II}=\frac{1}{2}+ \\ &+\frac{(1-\bar{x}_f)^2+(1+\gamma_0)\,\bar{x}_f^2}{1+\gamma_0\bar{x}_f}; \\ &\text{in region III } \bar{x}_{bo_2}\leqslant \bar{x}_f \leqslant 1.0, \\ &n=1.0,\ m=\frac{(1-\bar{x}_f)\,[3+2\gamma_0\bar{x}_f+\gamma_0(2+\gamma_0\bar{x}_f)]}{\bar{x}_f(1+\gamma_0)(3+2\gamma_0\bar{x}_f+\gamma_0)}, \\ &\Psi=\Psi_{III}=1-\frac{(1-\bar{x}_f)^2\gamma_0^2\,[2+\gamma_0(1+\bar{x}_f)]}{2(1+\gamma_0)\,[3+\gamma_0(1+2\bar{x}_f)]^2}. \end{aligned}\right\} \tag{26}$$

If one assumes that the instantaneous values of the true volumetric gas and vapor contents coincide with the corresponding instantaneous values of the volumetric flow-rate gas and vapor contents of the stream, i.e., that $\varphi(\tau) = \beta(\tau)$, then the time average of the true volumetric gas or vapor content of an unsteady two-phase stream is determined by the following expression:

$$\bar{\varphi} = \bar{\beta} = \frac{1}{T}\int_0^T \beta(\tau)\,d\tau, \tag{27}$$

where, in accordance with the standard definition,

$$\beta(\tau) = \frac{V_2(\tau)}{V_2(\tau)+V_1(\tau)} = \frac{G_2(\tau)}{G_2(\tau)+(1+\gamma_0)^{-1}G_1(\tau)}. \tag{28}$$

The expressions (27) and (28), with allowance for the results (24)-(26) obtained above on the minimization of the power of the stream and the well-known relationship of $\bar{x}_f$ with β_f

$$\bar{x}_f = \frac{\bar{\beta}_f}{1+\gamma_0(1-\bar{\beta}_f)}, \tag{29}$$

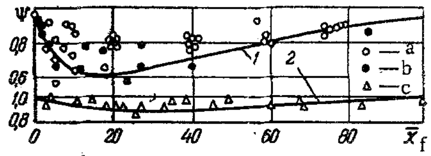

Fig. 1

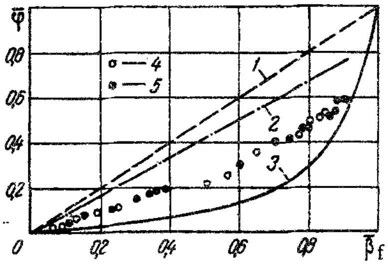

Fig. 2

Fig. 1. Dependence $\Psi = \Psi(\bar{x}_f, \bar{P})$ for irreversible pressure losses in local resistances for a steam–water stream: a) Janssen's experiments [14] with short inserts in a rectangular channel, $\bar{P} = 70$ bars, $\bar{w}_{cu} = (1.14\text{–}9.15)$ m/sec; b) experiments [30] for the entrance to a pipe from a collector, $\bar{P} = 60$ bars, $\bar{w}_{cu} = (0.25\text{–}1.5)$ m/sec; c) experiments [30] for entrance to a pipe from a collector, $\bar{P} = 180$ bars, $\bar{w}_{cu} = (0.25\text{–}1.5)$ m/sec; 1, 2) calculation by Eq. (25) for $\bar{P} = 65$ and 180 bars. $\bar{x}_f$, %.

Fig. 2. Dependence $\bar{\varphi} = \bar{\varphi}(\bar{\beta}_f)$ in a narrow washer cross sections: 1) calculation by steady homogeneous model $\bar{\varphi} = \bar{\beta}_f$; 2) by Armand's [4, 5] equation $\bar{\varphi} = 0.833\bar{\beta}_f$; 3) calculation by Eq. (30) for $\gamma_0 = 800$; 4), 5) experimental data of present work for subcritical and critical flow, respectively, of an air–water stream in a cylindrical washer of 40 mm inside diameter, 78 mm outside diameter, and 12 mm thick with $\bar{P} = (0.6\text{–}1.5)$ bars.

made it possible to obtain the following expressions for $\bar{\varphi}$ in the case of the laws (20) and (22) for the respective time variations in phase flow rates:

$$\bar{\varphi} = \frac{\bar{\beta}_f}{\bar{\beta}_f + (1+\gamma_0)^{1/3}(1-\bar{\beta}_f)}, \tag{30}$$

$$\left.\begin{aligned}
&\text{in region I } 0 \leqslant \bar{\beta}_f \leqslant \bar{\beta}_{bo_1} = \frac{(1+\gamma_0)\left[\sqrt{3(\gamma_0+1)(\gamma_0+3)} - 3\right]}{\gamma_0\left[1+\gamma_0+\sqrt{3(\gamma_0+1)(\gamma_0+3)}\right]},\\
&\bar{\varphi} = \bar{\varphi}_I = \left[3\,\frac{(\gamma_0+1)}{\gamma_0} - \bar{\beta}_f\right]\left[1 - \sqrt{1 - \frac{2\gamma_0\bar{\beta}_f}{3(1+\gamma_0)}}\right],\\
&\text{in region II } \bar{\beta}_{bo_1} \leqslant \bar{\beta}_f \leqslant \bar{\beta}_{bo_2} = \frac{3+2\gamma_0 - \sqrt{3(3+2\gamma_0)}}{2\gamma_0},\\
&\bar{\varphi} = \bar{\varphi}_{II} = \frac{\sqrt{\bar{\beta}_f}}{\sqrt{\bar{\beta}_f} + \sqrt{1-\bar{\beta}_f}},\\
&\text{in region III } \bar{\beta}_{bo_2} \leqslant \bar{\beta}_f \leqslant 1.0,\\
&\bar{\varphi} = \bar{\varphi}_{III} = 1 - \left[\frac{(\gamma_0+3)}{\gamma_0} - \bar{\beta}_f\right]\left[\sqrt{1 + \frac{2}{3}\gamma_0(1-\bar{\beta}_f)} - 1\right].
\end{aligned}\right\} \tag{31}$$

The theoretical calculation conducted fully predetermines the concrete physical pattern of flow of an unsteady two-phase stream.

In the case of the discontinuous change (20) in the phase flow rates with time a slug mode of flow of the two-phase mixture occurs in the entire range of vapor contents $0 < \bar{x}_f < 1.0$ with the successive alternation of liquid plugs and vapor slugs, since $x_1 = 0$ and $x_2 = 1.0$.

The duration of the existence of the vapor slugs and liquid plugs in the stream cross section under consideration varies with an increase in $\bar{x}_f$, shifting toward an increase in the former as $\bar{x}_f$ approaches 100%, since $\tau_2/\tau_1 = (1+\gamma_0)^{2/3} \cdot [\bar{x}_f/(1-\bar{x}_f)]$ according to (24).

In the flow of two-phase streams in pipes such a pattern is observed only in a comparatively narrow range of variation of $\bar{x}_f$ when the slug mode of flow occurs.

In the movement of a two-phase stream through local resistances of the type of thin throttle washers, sudden narrowings, or turns of the channel the unsteady process described above evidently can occur in almost the entire region of variation of $\bar{x}_f$. In fact, in this case stagnant vortex zones of very considerable

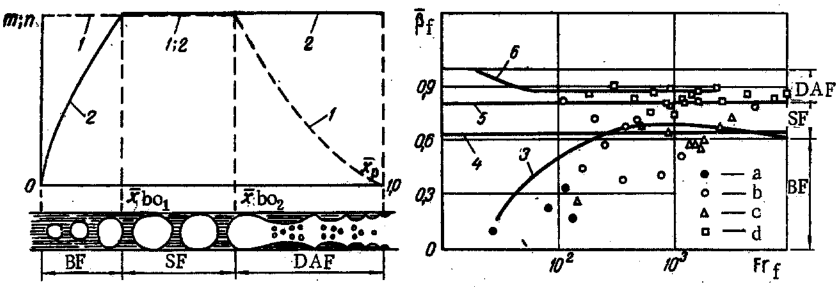

Fig. 3. Modes of flow of a two-phase stream in a pipe with a harmonic law of oscillations of the phase flow rates in the stream: BF, SF, DAF) bubble, slug, and disperse-annular modes of flow; 1) amplitude m; 2) amplitude n; 3) BF–SF boundary according to Baker [29]; 4) $\bar{\beta}_{bo_1}$ for $\gamma_0 = 20$; 5) $\bar{\beta}_{bo_2}$ for $\gamma_0 = 20$; 6) SF–DAF boundary according to Haberstroh and Griffith [14]; a, b, c, d) BF, SF, emulsion flow and DAF according to experiments of Bergles and Hsieu [14] in a pipe with $\bar{P} = 70$ bars. $Fr_{mi} = \bar{w}^2_{mi}/gd$.

size form ahead of the washer or local narrowing or in the region of the outer radius of a turn in the channel. Because of the high inertia of the liquid and its predominent movement along the channel walls considerable masses of liquid can concentrate in these zones for any vapor contents of the two-phase stream. Periodic surges of considerable amounts of liquid into the main core of the stream, which are capable of completely covering the narrow cross section of the local resistance for a short time, are possible under the effect of the gas stream. Moreover, at the times of accumulation of liquid in the stagnant zone the lighter and more mobile gaseous phase predominantly moves through the narrow cross section.

In connection with this, the theoretical results (25) and (30) for Ψ and $\bar{\varphi}$, obtained for oscillations of the rectangular pulse type (20) in the flow rates of the phases in the stream, are compared in Figs. 1 and 2 with experimental data for the movement of two-phase streams through local resistances of the type indicated above.

The sinusoidal oscillations G_1 and G_2 of (22) correspond to a different structure and dynamics of the development of a two-phase stream with an increase in $\bar{x}_f$. This conclusion follows from an analysis of Eq. (26) for the relative amplitudes n and m of the oscillations of the phase flow rates in different regions of variation in the vapor content $\bar{x}_f$.

The character of the variation in n and m with an increase in $\bar{x}_f$ from 0 to 1.0 is presented in Fig. 3.

Let us analyze in more detail, for example, region I of low vapor contents $0 < \bar{x}_f < x_{bo_1}$, where the average liquid flow rate $\bar{G}_1 = G_{10}$ is predominantly much higher than the average vapor (gas) flow rate $\bar{G}_2 = G_{20}$. The relative amplitudes n of the oscillations in the liquid flow rate in this region are slight, whereas the relative amplitudes m of the oscillations in the vapor flow rate are maximal, since $m = 1.0$ in the entire region of variation $0 < \bar{x}_f < \bar{x}_{bo_1}$.

Periodically, when $\sin \omega\tau = 1.0$, a fixed cross section of the stream is fully covered over by liquid without vapor (gas) inclusions ($G_2 = 0$). At the other times $G_1 > 0$ and $G_2 > 0$ in this stream cross section, i.e., vapor and liquid are present at the same time. Such a process occurs when isolated vapor (or gas) bubbles are present in a continuous liquid stream, i.e., with a bubble mode of flow of a two-phase stream in pipes and channels.

A similar analysis leads to the conclusion that in region II $\bar{x}_{bo_1} \le \bar{x}_f \le \bar{x}_{bo_2}$ there is a slug mode of flow of a two-phase stream, while in region III $\bar{x}_{bo_2} < \bar{x}_f \le 1.0$ there is a disperse-annular wave mode of flow with emergence into pure vapor.

The replacement of the bubble mode of flow by the slug mode and of the slug mode of flow by the disperse-annular wave mode occurs at the limiting values $\bar{x}_{bo_1}$ and $\bar{x}_{bo_2}$ of the flow-rate vapor content.

This sequence of change of the structure of the two-phase stream under consideration with an increase in the vapor content $\bar{x}_f$ is presented on the left side of Fig. 3 in the form of the conventional flow patterns of this stream corresponding to certain values of $\bar{x}_f$.

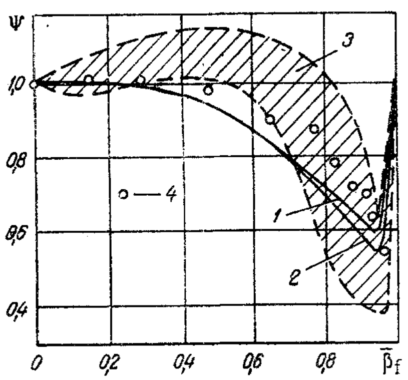

Fig. 4

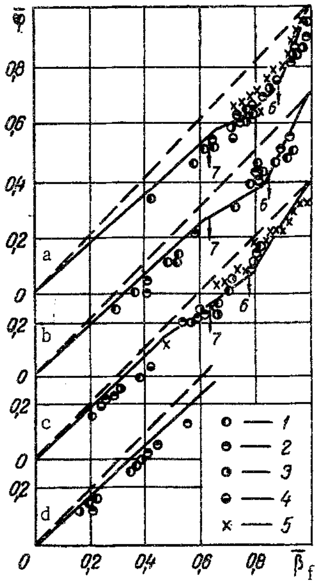

Fig. 5

Fig. 4. Dependence $\Psi = \Psi(\overline{\beta}_f)$ for horizontal pipes (air–water): 1, 2) calculation by Eqs. (26) for γ_0 = 400 and 800; 3) region of experimental values for $\overline{P} \approx 1$ bar from data of [9]; 4) authors' experiments, $\overline{P}$ = 2 bars, $\overline{w}_0$ = 1 m/sec, l/d = 30; l_{stab} = 300d, d = 20 mm.

Fig. 5. Dependence $\overline{\varphi} = \overline{\varphi}(\overline{\beta}_f, \gamma_0)$ for flow of a steam–water stream in annular slots (1-4) [26] and pipes 5 [27]: a-d) $\overline{P}$ = 20, 40, 70, and 100 bars or γ_0 = 86, 40, 20, and 12, respectively; dashed curve: calculation by steady homogeneous model; solid line: by Eqs. (31); 1-4) $\overline{w}_0$ = 0.5, 1.0, 2.0, and 4.0 m/sec; 7) β_{bo_1}; 6) β_{bo_2}.

As is known, a similar structure of a two-phase stream with the corresponding replacement of the modes occurs in the pressurized flow of two-phase streams in straight pipes and channels of constant cross section [11, 14, 25].

In connection with this the theoretical results (26) and (31) for Ψ, $\overline{\varphi}$, and the boundaries $\overline{\beta}_{bo_1}$ of the modes of flow (31), obtained in an analysis of sinusoidal oscillations of the phase flow rates with time, are compared in Figs. 3-5 with experimental hydraulic characteristics measured in the flow of two-phase streams in channels and pipes.

The results of these comparisons must be taken as positive, not only in a qualitative, but also in a quantitative, respect.

In summing up, one must conclude that the proposed unsteady model of a two-phase mixture quite satisfactorily describes the basic hydraulic characteristics of a real, high-velocity, incompressible, stabilized, adiabatic, two-phase stream.

LITERATURE CITED

1. S. G. Teletov, Dokl. Akad. Nauk SSSR, 50, Novaya Ser. (1945).
2. R. C. Martinelli and D. B. Nelson, Trans. Amer. Soc. Mech. Eng., 70, 695-702 (1948).
3. S. S. Kutateladze, Sov. Kotloturbostr., No. 2 (1946).
4. A. A. Armand and E. I. Nevstrueva, Izv. Vses. Teplotekh. Inst., No. 2 (1950).
5. A. A. Armand, in: Hydrodynamics and Heat Exchange during Boiling in High-Pressure Boilers [in Russian], Izd. Akad. Nauk SSSR, Moscow (1955).
6. S. G. Teletov, in: Hydrodynamics and Heat Exchange during Boiling in High-Pressure Boilers [in Russian], Izd. Akad. Nauk SSSR, Moscow (1955).
7. S. S. Kutateladze and M. A. Styrikovich, Hydraulics of Gas–Liquid Systems [in Russian], Gos. Énerget. Izd., Moscow (1958).

8. M. Sil'vestri, Problems of Heat Exchange [in Russian], Atomizdat, Moscow (1967).
9. S. G. Teletov, Tr. Tsentr. Kotloturb. Inst., Leningrad, No. 59 (1965).
10. O. M. Baldina, V. A. Lokshin, D. F. Peterson, and A. L. Shvarts, Tr. Tsentr. Kotloturb. Inst., Leningrad, No. 59 (1965).
11. L. S. Tong, Boiling Heat Transfer and Two-Phase Flow, Wiley, New York (1965).
12. Normative Method of Hydraulic Calculation of Steam Boilers [in Russian], Vol. 1, Part 33, Tsentr. Kotloturb. Inst., Leningrad (1973).
13. S. S. Kutateladze, Boundary Turbulence [in Russian], Izd. Nauka, Sibirsk. Otd. Akad. Nauk SSSR, Novosibirsk (1973).
14. V. M. Borishanskii (editor), Advances in the Field of Heat Exchange [Russian translation], Mir, Moscow (1970).
15. S. S. Kutateladze, A. P. Burdukov, V. E. Nakaryakov, Yu. V. Tatevosyan, and V. A. Kuz'min, Dokl. Akad. Nauk SSSR, 200, No. 21 (1971).
16. V. P. Bobkov, M. Kh. Ibragimov, and V. I. Subbotin, Inzh.-Fiz. Zh., 20, No. 4 (1971).
17. N. Miller and R. E. Mitchell, J. Brit. Nucl. Energy Soc., 9, No. 2, 94-100 (1970).
18. Shigeru Hinata, Bull. Jap. Soc. Mech. Eng., 15, No. 88, 1228-1235 (1972).
19. K. Nishikawa, K. Sekoguchi, and T. Fukano, Bull. Jap. Soc. Mech. Eng., 12, No. 54 (1969).
20. G. V. Tsiklauri, Teplofiz. Vys. Temp., 10, No. 6 (1972).
21. T. Dallavalle, T. Rossini, and G. Vanoli, Energia Nuclear, 10, No. 6, 332-342 (1973).
22. F. J. Moody, Trans. Amer. Soc. Mech. Eng., J. Heat Transfer, 87, No. 1, 134 (1965).
23. Zivi, Teploperedacha, No. 2, 139 (1964).
24. J. E. Gruver and R. W. Moulton, Amer. Inst. Chem. Eng. J., 13, No. 1, 52-60 (1967).
25. S. S. Kutateladze (editor), Investigation of Turbulent Flows of Two-Phase Media [in Russian], Izd. Inst. Teplofiz. Akad. Nauk SSSR, Novosibirsk (1973).
26. A. G. Lobachev, É. A. Zakharova, B. A. Kol'chugin, G. G. Kruglikhina, and D. A. Labuntsov, Heat and Mass Transfer, Collection of Proceedings of Fourth All-Union Conference on Heat and Mass Exchange [in Russian], Vol. 2, Part 1, Minsk (1972), p. 299.
27. Z. L. Miropol'skii and R. I. Shneerova, Teplofiz. Vys. Temp., 1, No. 1 (1963).
28. D. U. Merdok, Teor. Osnovy Inzh. Raschet. 84, No. 4 (1962).
29. O. Baker, Oil Gas J., 53, 185-190 (1954).
30. S. I. Mochan, in: Problems of Heat Transfer and the Hydraulics of Two-Phase Media [in Russian], Gos. Énerget. Izd., Moscow–Leningrad (1961).
31. I. I. Morozov and P. P. Vasil'ev, Teploénergetika, No. 1 (1968).